2 - Transformer Basics

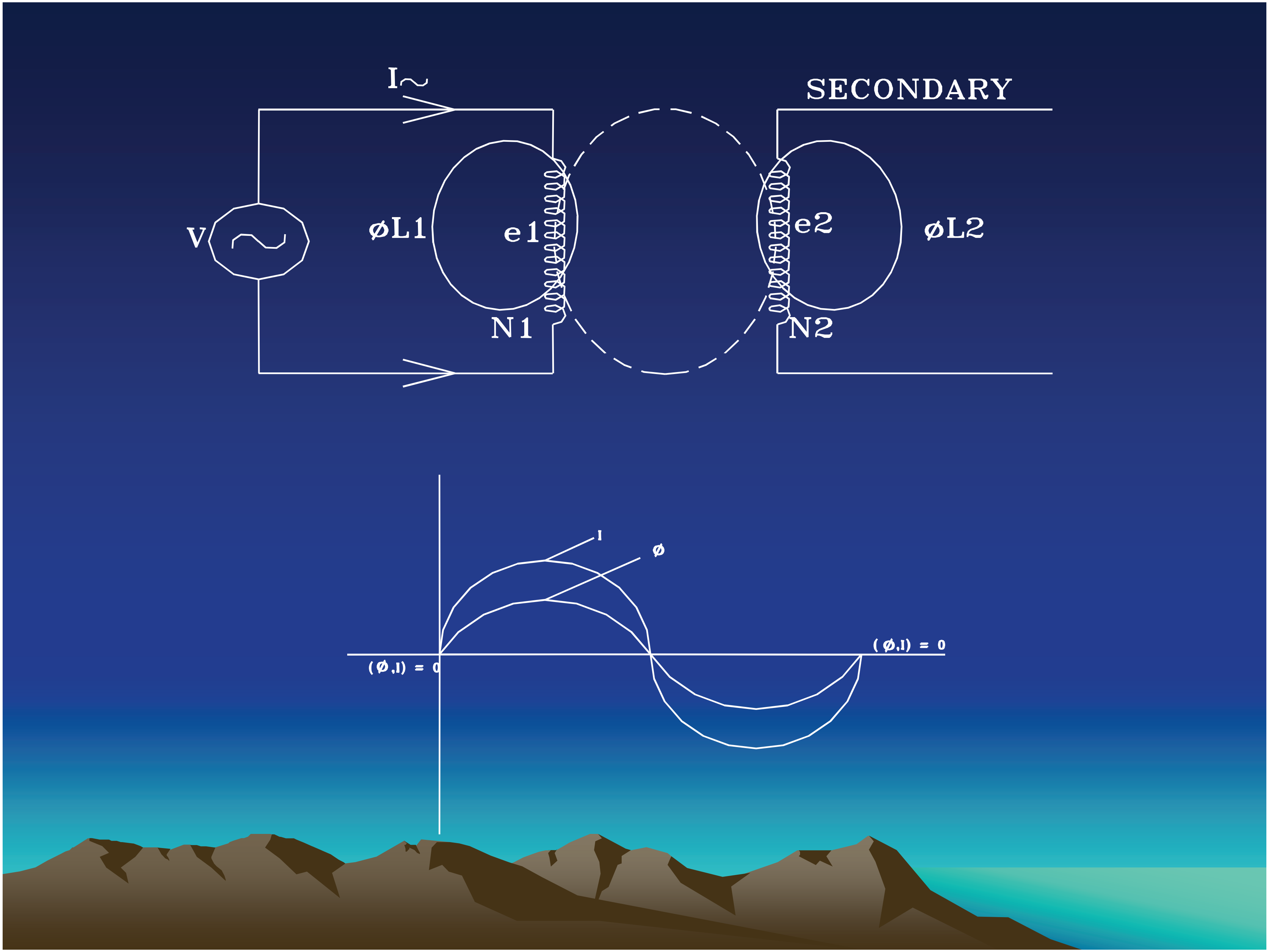
مبادىء نظرية

TRANSFORMERS

Definition :

Transformers can be defined as a static electric machine which converts electric energy from one potential to another at the same frequency .

- It can also be defined as consists of two electric circuits linked by a common variable flux.


I~
SECONDARY
V
øL1
e1
N1
e2
øL2
N2
I
ø
(ø,I) = 0
(ø,I) = 0

Theory of operation :

The primary coil of the transformer is connected to a supply of sine wave voltage . an alternating sine wave current will flow in the primary . thus the primary m.m.f (N.I) will produce a common

flux (g) which is also alternating and in phase with the current according to Faraday′s law the common flux interesting two coils will induce in them an alternating e.m.f (e1 , e2) .

$$e_1 = -\frac{N_1 d\phi}{dt} \qquad (1)$$

e1 is an e.m.f of self induction

e2 is an e.m.f of Mutual induction

$$e_1 = -N_2 \frac{d\phi}{dt} \qquad (2)$$

from 1,2 ∴ the transformation ratio

$$K = \frac{e_1}{e_2} = \frac{N_1}{N_2}$$

Applying Kirchoff's law on the primary circuit.

$V = - e1$

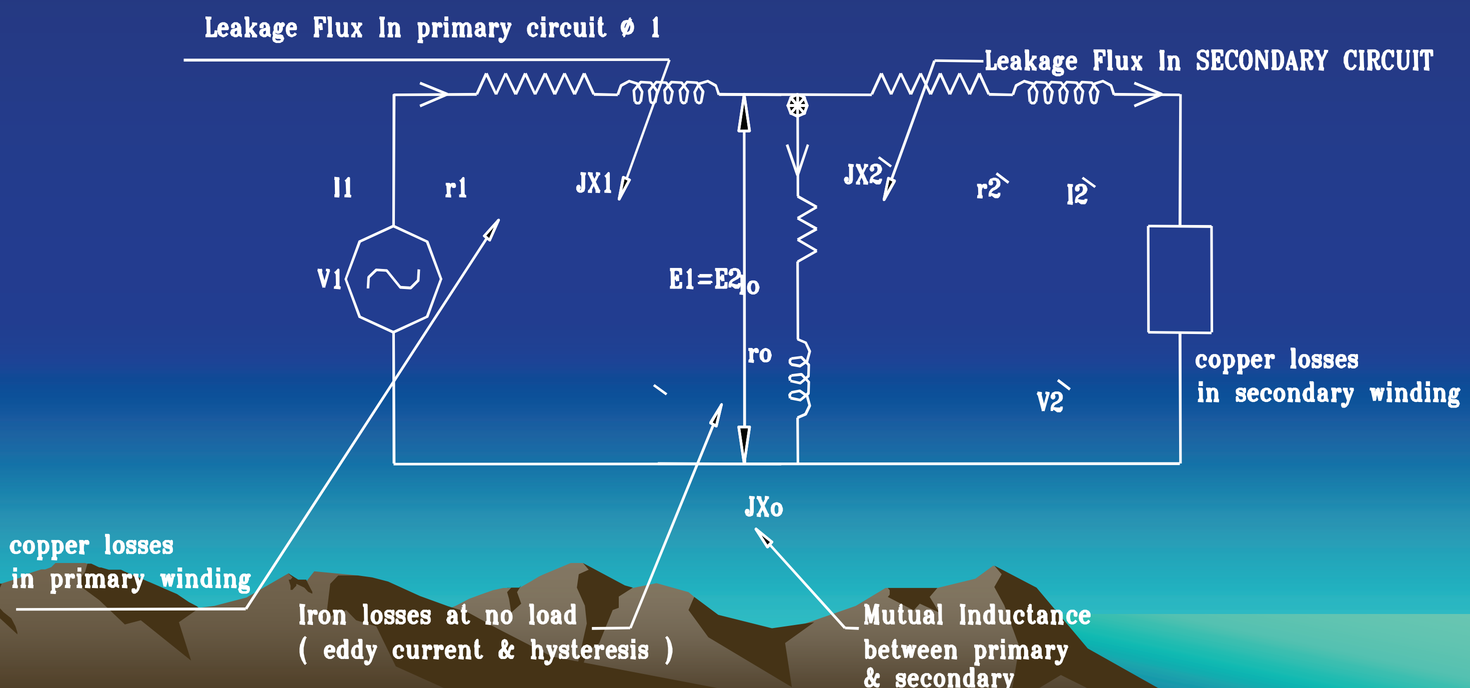
$V + e1 = 0$

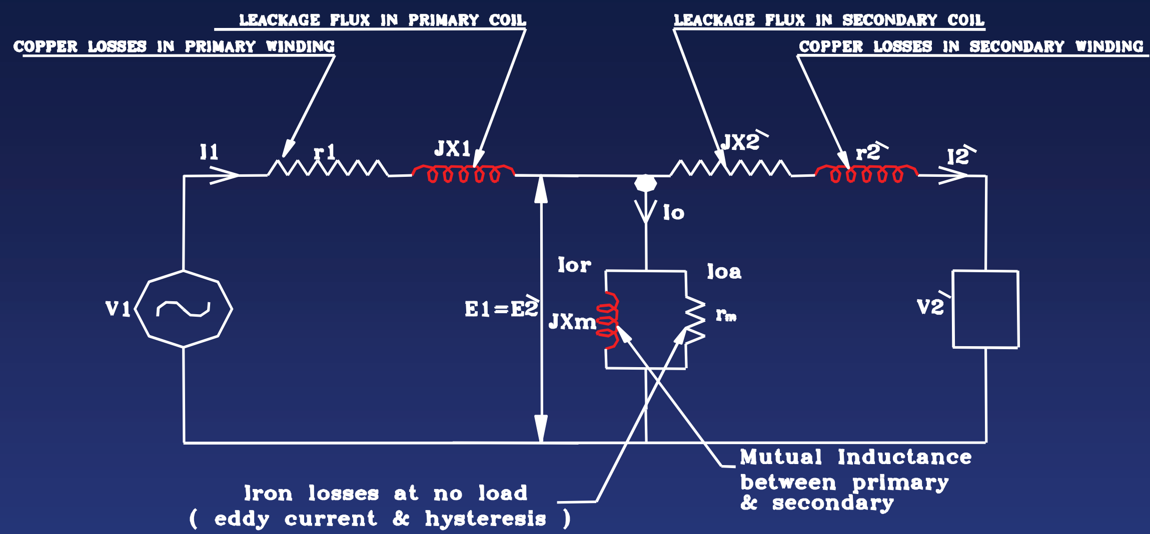
$$\sum e.m.f = \sum V.D$$

From the secondary circuit $e2 = v2$

$$K = \frac{e_1}{e_2} = \frac{N_1}{N_2} = \frac{V_1}{V_2}$$

- ***<u>Equivalent circuit :</u>***

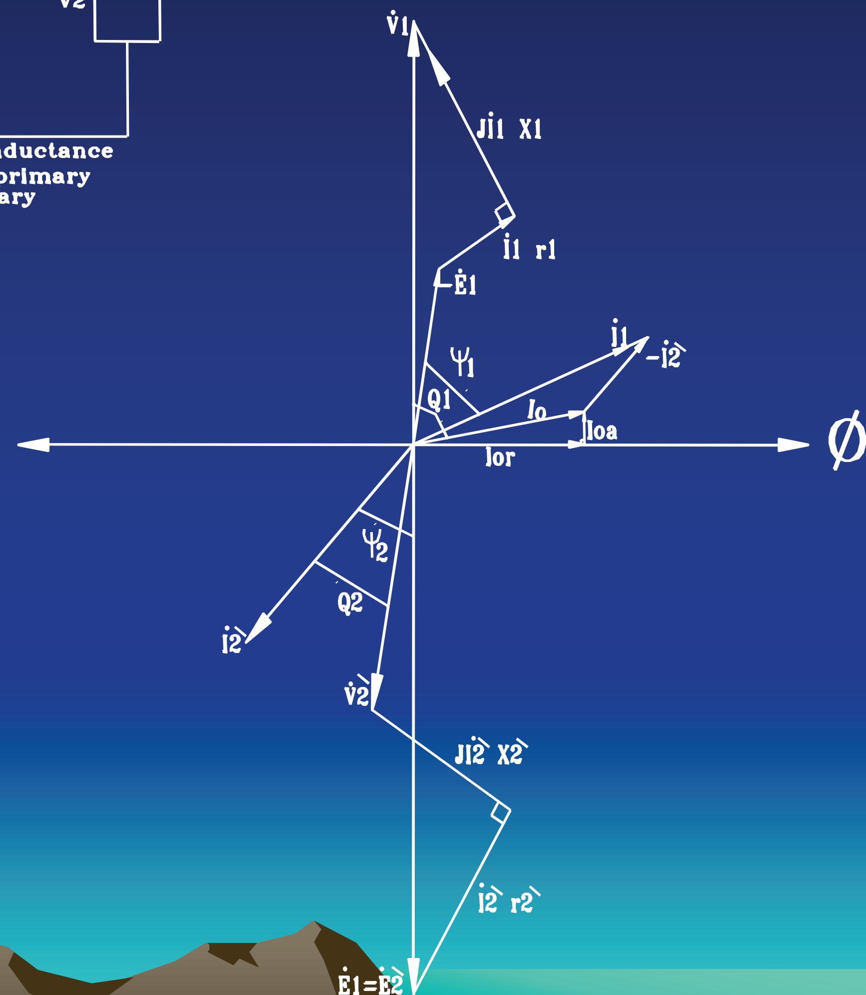




Io = 10 : 15 % of rated current .

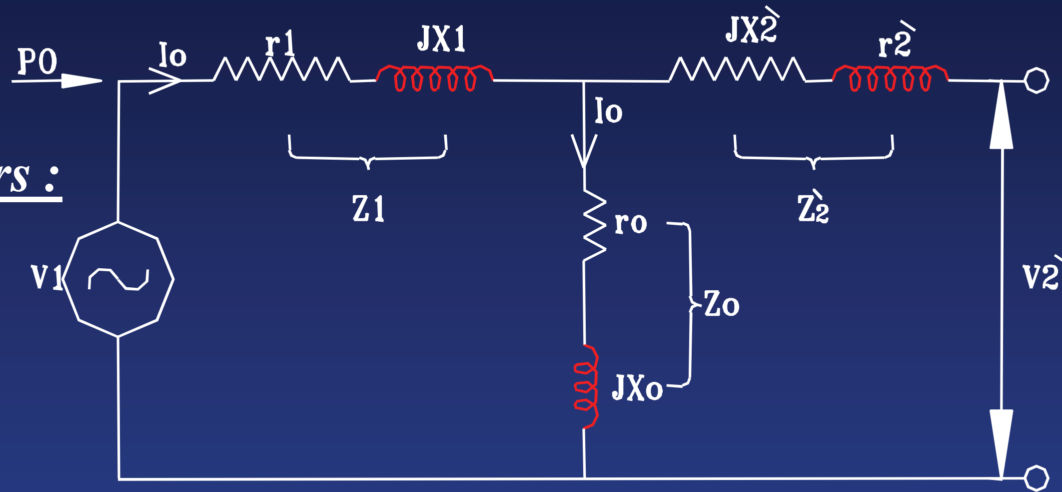
$$\dot{V}_1 = -\dot{E}_1 + \dot{I}_1 r_1 + J\dot{I}_1 X_{11}$$

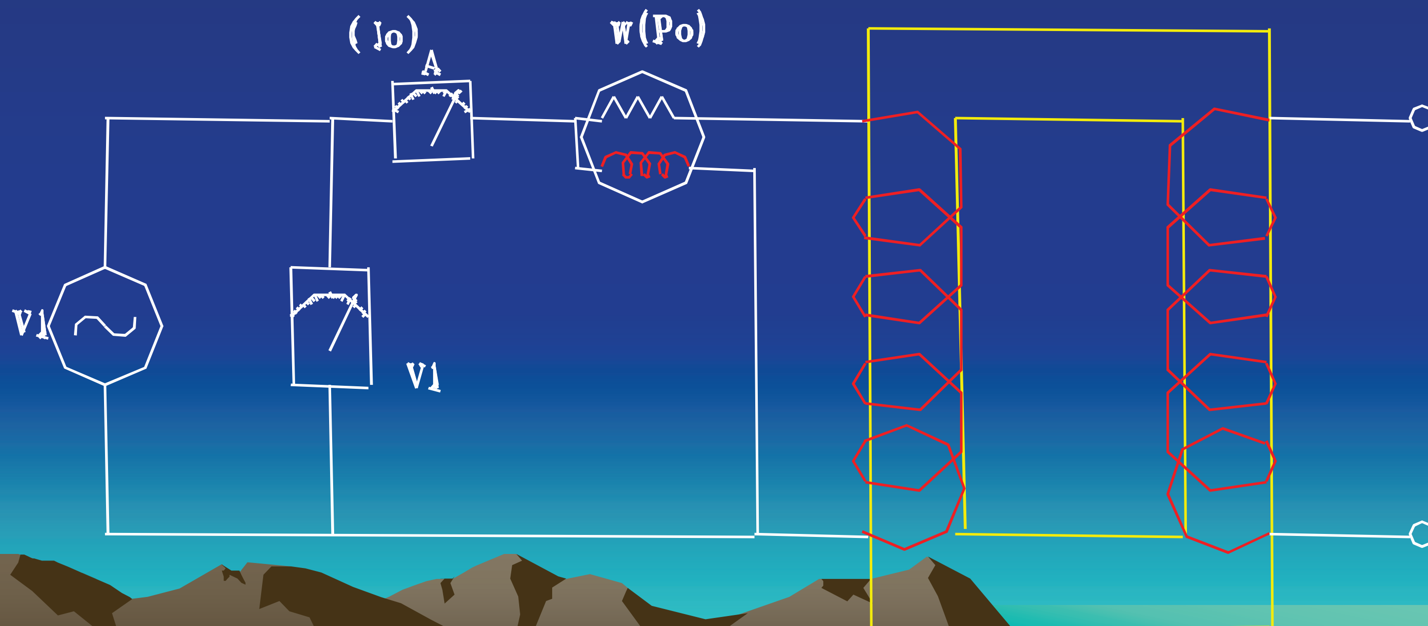
$$V'_2 = -\dot{E}'_2 + I'_2 r'_2 + JI'_2 X'_2$$



Tansformer testing :

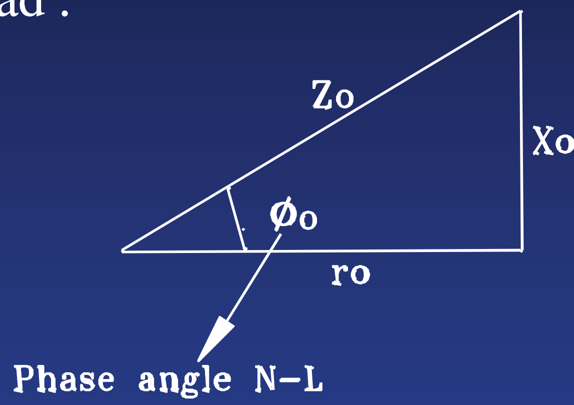
Determination of parameters :





Connect the primary to a source of alternating current at nomial voltage the secondary is open circuit – read the magnitude of (Io , V1 , Po) at no load .

The impeadence of the circuit at no load .



$$\frac{V_1}{I_o} = |Z_1 + Z_0|$$

$$\because Z_1 << Z_o$$

$$Z_0 \cong \frac{V_1}{I_o}$$

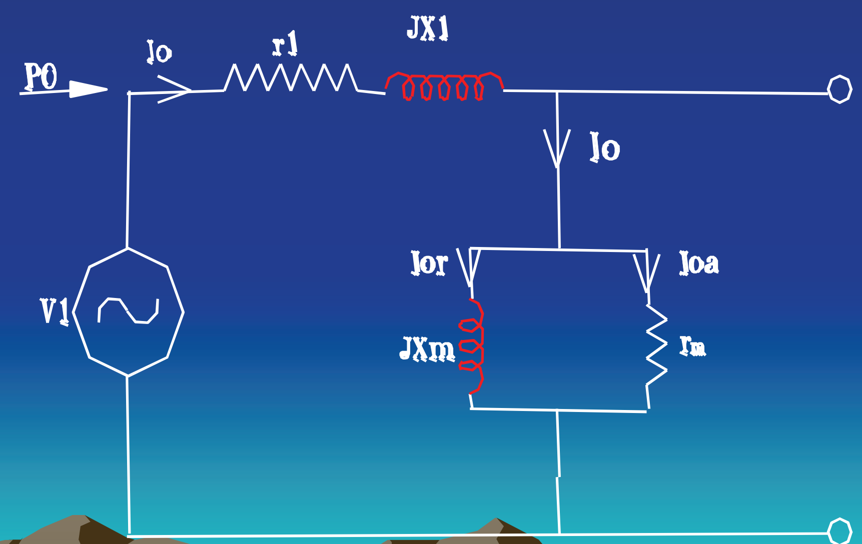
$$\because Z_1 << Z_o$$

Z1 can be neglected

$$r_o = \frac{p_o}{(I_o)^2}$$

$$|Z_o| = \sqrt{r^2{}_o + X_o{}^2}$$

$$x_o = \sqrt{|Z_o|^2 - r_o{}^2}$$



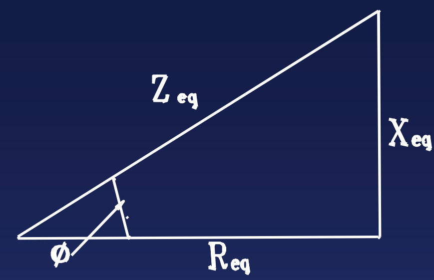
-Another method:

Po = V1 Io is go

$$|Z_o| \cong \frac{V_1}{I_0}$$

$$\cos\ \phi_o \cong \frac{Po}{V_1 Io} \qquad \phi_o = \cos^{-1}\frac{Po}{V_1 Io}$$

$$Ro = |Z_o|\ \cos\phi_o$$



For parallel circuit rm & JXm :

Neglect *Zm* relation to Z1

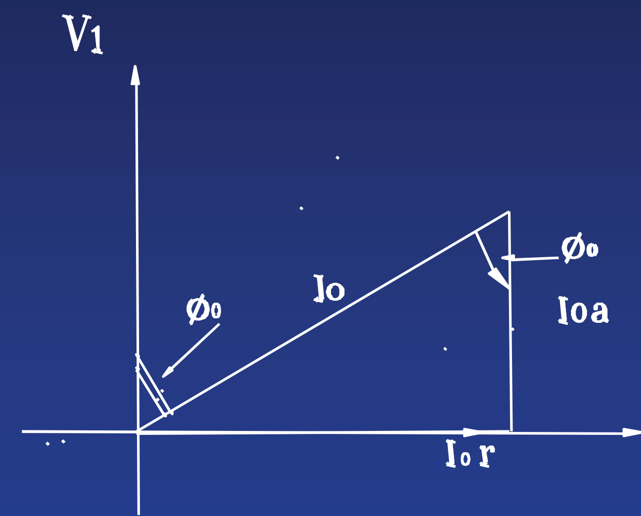
$$x_o = |Z_o|\ \sin\phi_o$$

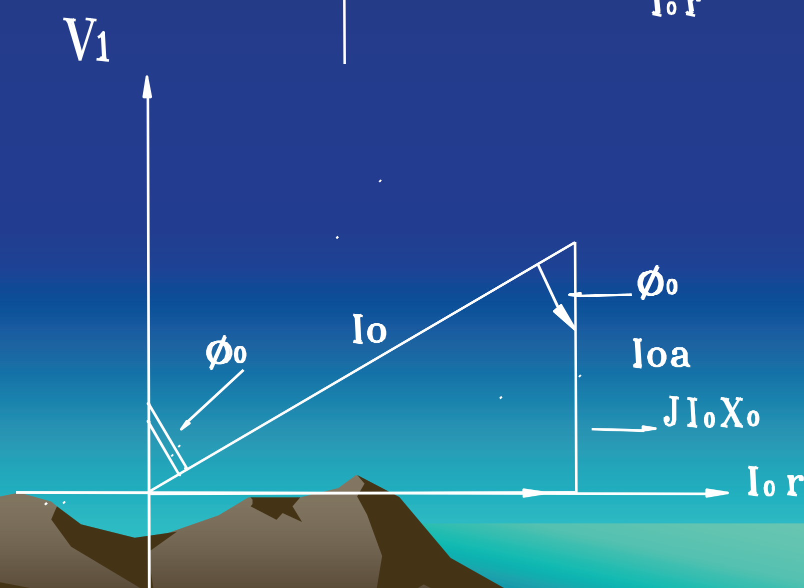
$$I_o a = I_o\ \cos\ \phi_o$$

$$I_o r = I_o\ \sin\ \phi_o$$

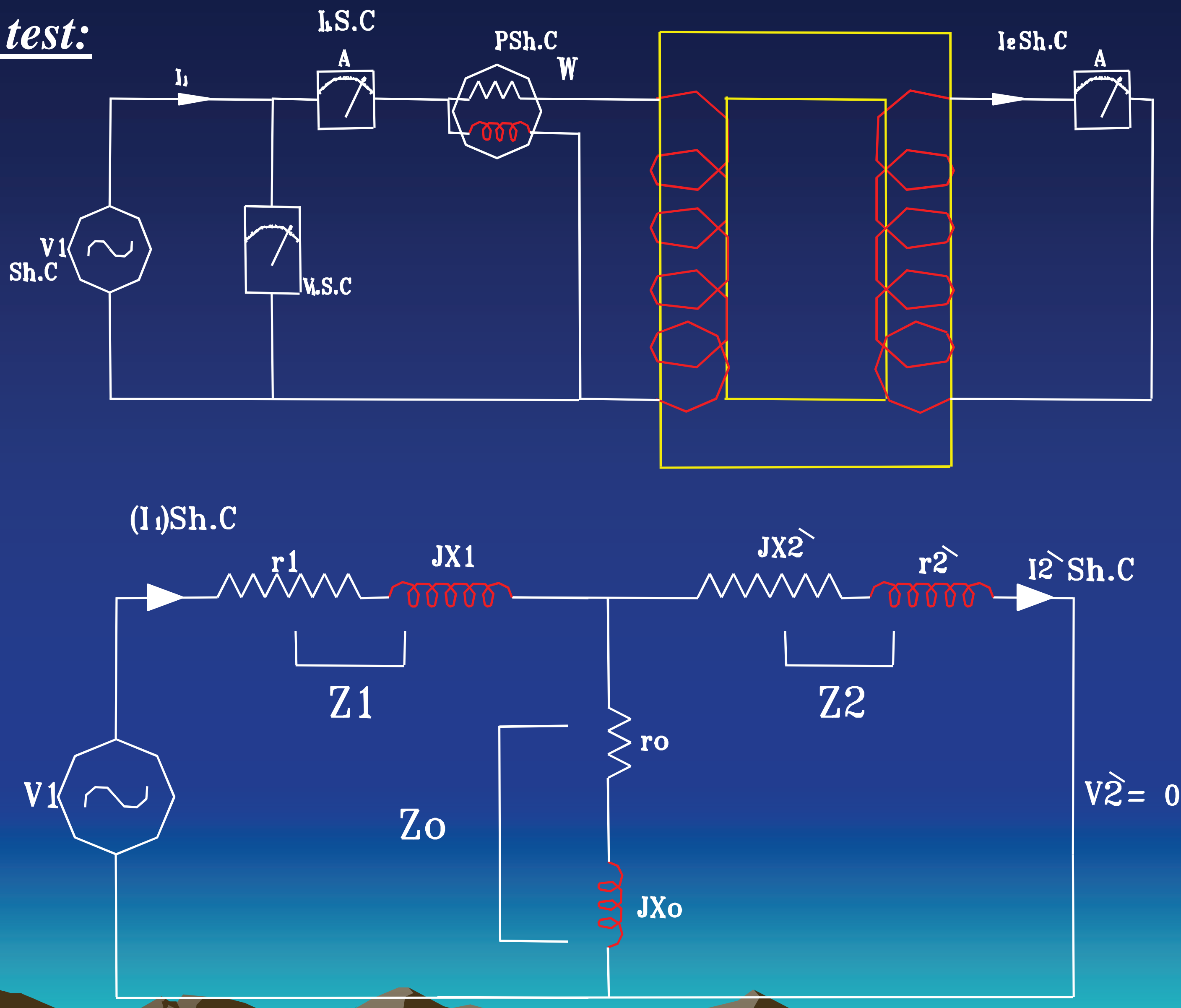
$$\phi_o = \cos^{-1}\left(\frac{P_o}{V_1}\right)$$

$$r_{m2} = \frac{V_1}{Ioa} \qquad X_m = \frac{V_1}{Ior}$$





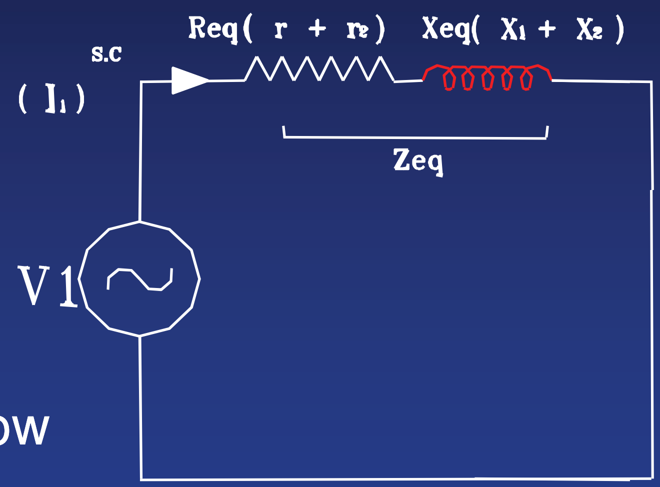
-Short circuit test:



Connect the primary to a reduced voltage (from 15 – 20 % of V1) until the primary current becomes near to the value of the full load current of the primary .

Short circuit the secondary winding .

Measure ; (V1) sh.c (I1) sh.c (P1) sh.c



In the circuit Z'_2 , $Z'o$ are connected in parallel
Zo is of the order of (10)$^{-2}$ relative to Z'_2

So the effect of Zo can be simplified to the show figure (c) .

$$\frac{V_1 sh.c}{I_1 sh.c} = |Z|eq$$

Where ;

Zeq = Req + Jxeq

Req = r1 + r2′

$$x_{eq} = x_1 + x'_2$$

$$P1\ (sh.c) = (V1)sh.c\,(I_1)sh.c\,(\cos\phi)\,sh.c$$

$$\phi\ sh.c = \cos^{-1}\frac{P_1}{\left(\frac{P_1}{V_1 I_1}\right)sh.c}$$

$$x_{eq} = |Z_{eq}|\sin\phi$$

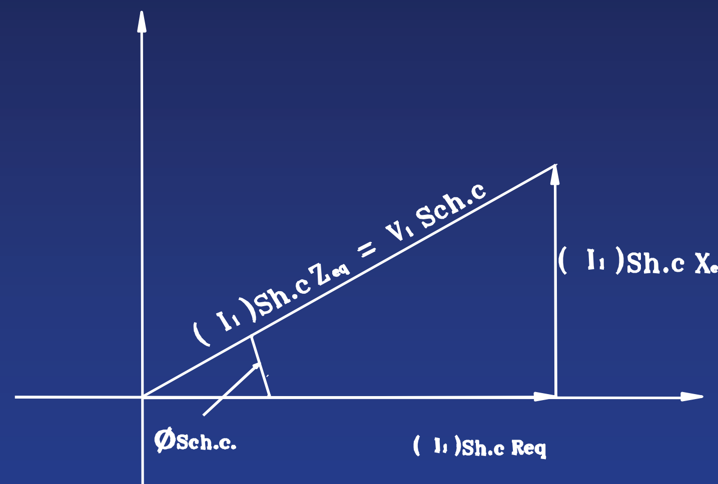
$$R_{eq} = |Z_{eq}|\cos\phi$$

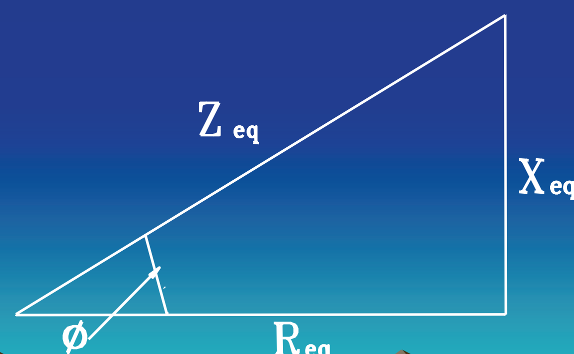
$$r_1 \cong r'_2$$

$$X_1 \cong X'_2\ \frac{X_{eq}}{2}$$

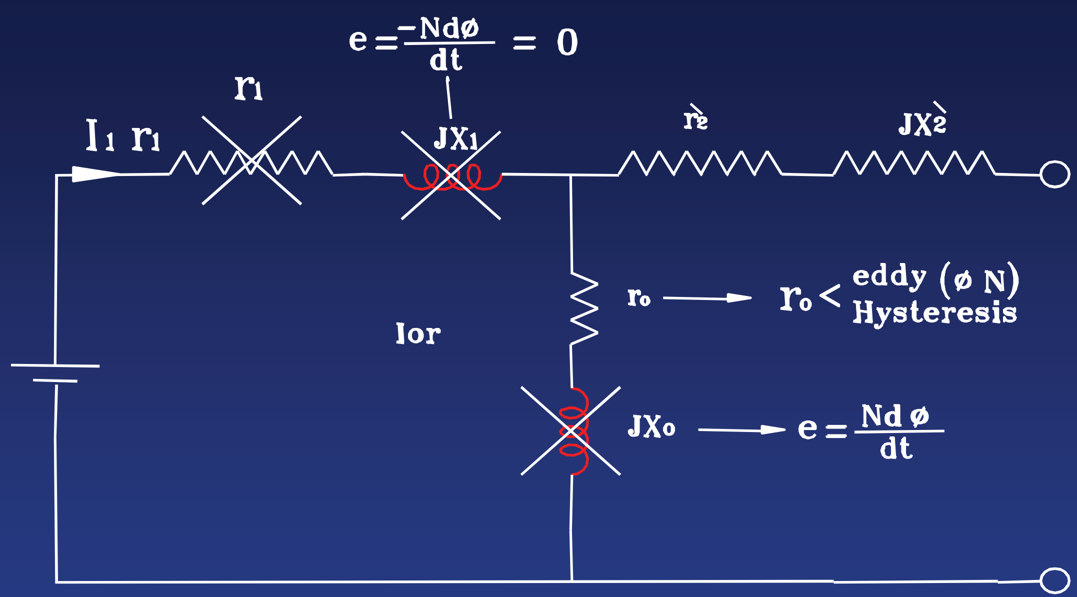
$$r_2 = \frac{r'_2}{K^2} \qquad x_2 = \frac{x'_2}{k^2}$$

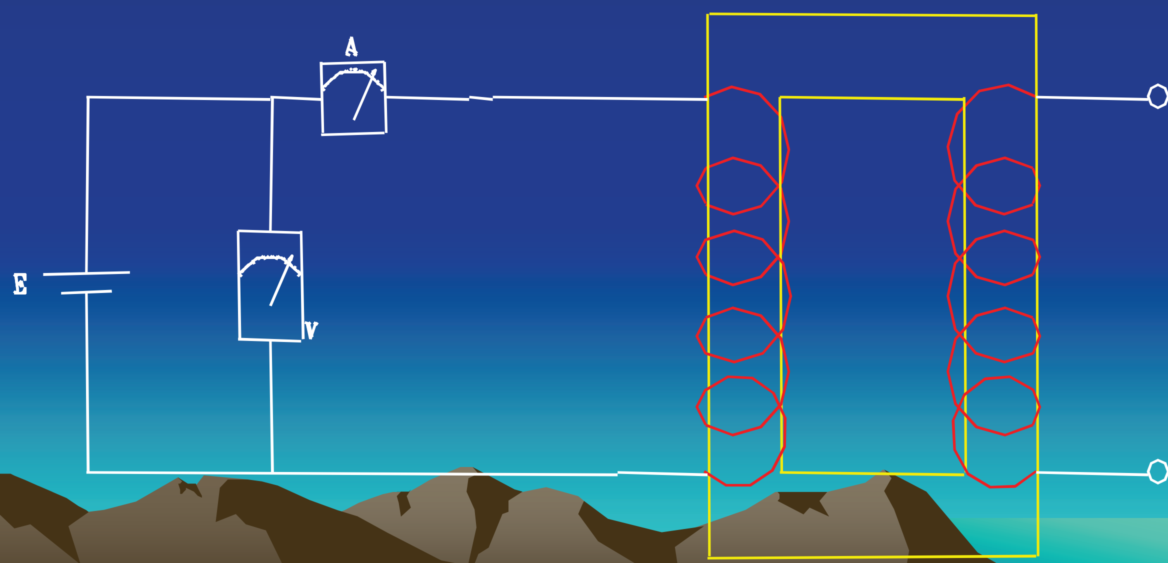
$(I_1)Sh.c\ Z_{eq} = V_1\ Sch.c$





•D.C Test :


e = −Ndø/dt = 0
r1
I1 r1
JX1
r2`
JX2`
r0
r0 < eddy (ø N)
Hysteresis
Ior
JX0
e = Nd ø/dt


A
E
V

Connect the primary coil with a direct current supply . measure the applied voltage and the current .

$$\frac{E}{I} = r_1 \ \text{ohms}$$

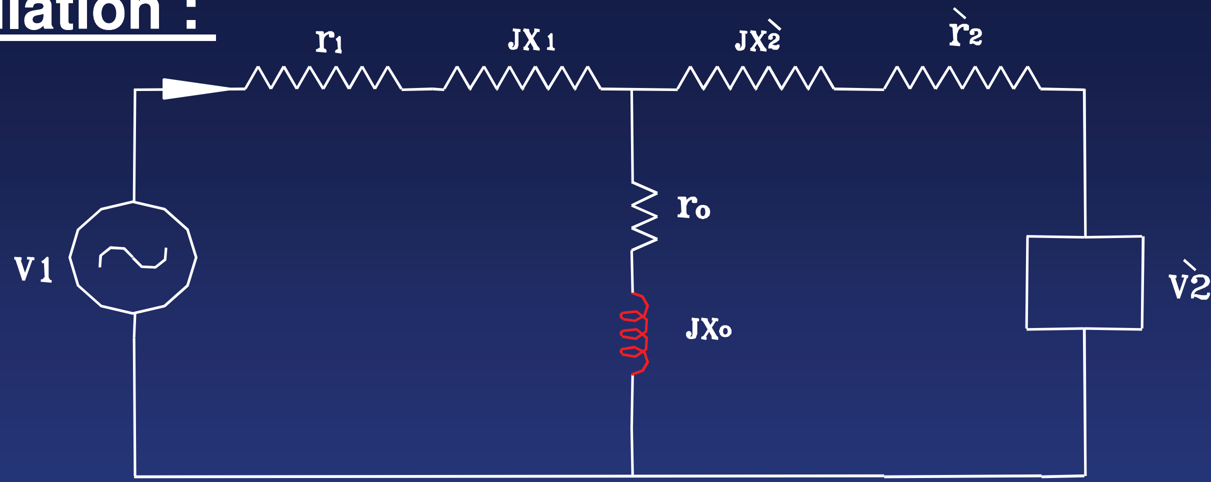
The effect of X1 , X_0 will not appear when using direct current

$$\left[\ X \to \frac{Ldi}{dt} \right]$$

I is the const. relative to time] also the effect of ro will not appear because it represent the eddy and hysteresis losses which are not existing in the case of direct current they appear only when there is varialable flux in the core .

Similarly we can determine the resistance of the secondary (r2) by connecting the battery to the terminals of the secondary coil .

Voltage regulation :



The voltage regulation is defined as the change in the secondary voltage of a loaded transformer when the load is removed . while the primary voltage is constant at it′s nominal value .

$E = (V_2)_{n.l} - V_2 \text{ load}$

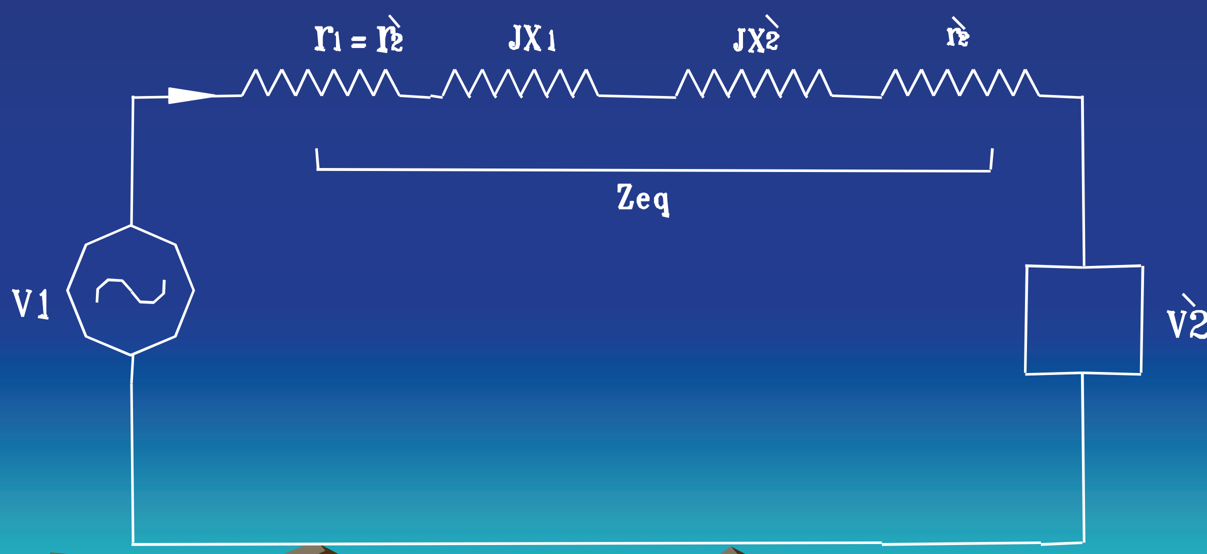
In order to enable the comparison between transformers of different working voltages , the voltage regulation in usually expected as percent or a per unit value related to the secondary voltage at load .

usually the voltage regulations is determined for full load conditions . so to simplify equivalent circuit

$$\varepsilon_{V.R} = \frac{(V_2)_{n.L} - (V_2)_L}{(V_2)_L} \times 100 = \quad \%$$

$$\varepsilon_{V.R} = \frac{(V_2)_{n.L} - (V_2)_L}{(V_2)_L} \quad = \text{per unit}$$

calculation the effect of I0 is neglected and we get the following simplified equivalent circuit and the corresponding to it vector diagram (Kapp vector diagram)



• Io neglected :

$I_1 = I'_2$

$Z_q = Req + JX_{eq} = (r_1 + r'_2) + J(X_1 + X'_2)$

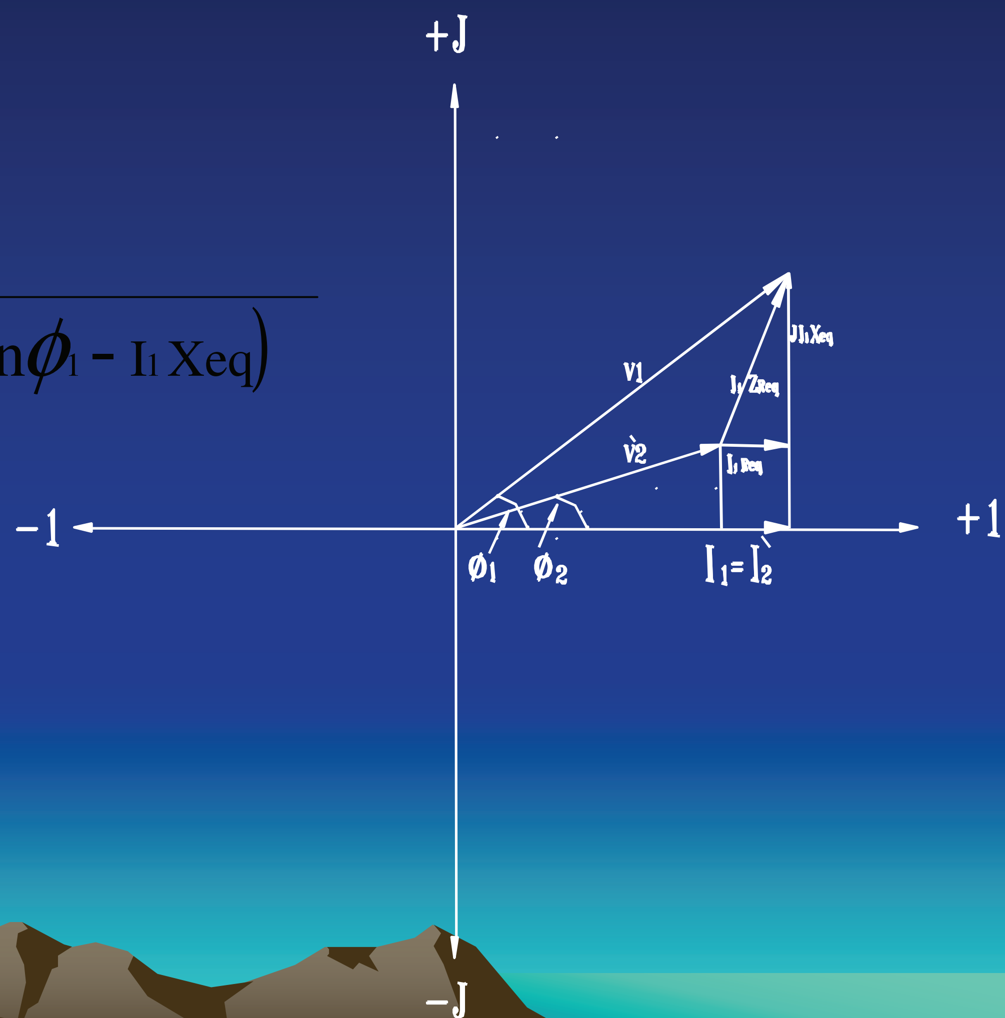
To calculate the voltage regulation the following value must be determined .

$V_1, I_1 \cos g_1$ and Z_{eq} .

$$\dot{V}_1 = \dot{V}'_2 + I1_{Req} + J\dot{I}_1 X_{eq}$$

$$\dot{V}'_2 = \dot{V}_1 + I_1 R_{eq} + J\dot{I}_1 X_{eq}$$

$$\dot{V}_2 = \sqrt{\left(V_1 \cos\phi_1 - I_1 Req\right)^2 + \left(V_1 \sin\phi_1 - I_1 Xeq\right)}$$



Note:

$(V'_2)_{n.1} = V_1$

$$\varepsilon = \frac{(V_2)n.l - (V_2)l}{(V_2)l}$$

$$\varepsilon = \frac{(K\,V_2)n.l - (KV_2)l}{(KV_2)l}$$

$$\varepsilon = \frac{(V'_2)n.l - (V'_2)l}{(V'_2)l} = \frac{V_1 - V'_2}{V'_2}$$

where V′2 is calculated by eq (1)

some times the quantities known are V2 , I2 Cos **g**2 , Zeq
in this case To find the voltage regulation V1 we can calculated from the geometry as

<u>Note :</u>

Parameters r′2 , X′2 , I′2 , E′2 as follows :

$$\dot{V}_1 = \sqrt{\left(V'_2\cos\phi_2 - I'_2R_{eq}\right)^2 + \left(V'_2\sin\phi_2 - I'_2Xeq\right)}$$

Where ;

$$I'_2 = \frac{I_2}{K}$$

r′2 = k2 r2 , x′2 = k2 x2

E′2 = kE2

$$k = \frac{N_1}{N_2} = \frac{E_1}{E_1} \cong \frac{V_1}{V_2} \cong \frac{I_2}{I_1}$$

Transformer efficiency (η) :

$$\eta = \frac{P_{output}}{P_{input}} = \frac{P_{in\,-\,\text{losses}}}{P_{in}}$$

$$\therefore\ P_{cu} \propto I^2 \qquad\qquad p_{cu} = I_1^2\left(r_1 + r_2'\right) = I_1\,\text{req} = I_2'^2\,\text{req}$$

This means that if the cu losses are known at a certain load (current) , then the copper losses can be determined at another load .

$$\frac{\left(P_{cu}\right)_a}{\left(P_{cu}\right)_b} = \frac{\left(I^2\right)_a}{\left(I^2\right)_b}$$

I1 : nominal value (full load value) usually the copper losses are determined from a short circuit test at a current equal to the full load or nominal value , accordingly the equation can be written as :

$$\frac{P_{cu}}{(P_{cu})_{f.l}} = \frac{(I)^2}{(I)^2_{f.l}}$$

$$(p_{cu})_{required} = (p_{cu})_{f.l}\left(\frac{I}{I_{f.l}}\right)^2$$

$$let \quad \frac{I}{I_{f.l}} = X$$

$$P_{cu}\ \text{required} = X^2\,(P_{cu})_{f.l}$$

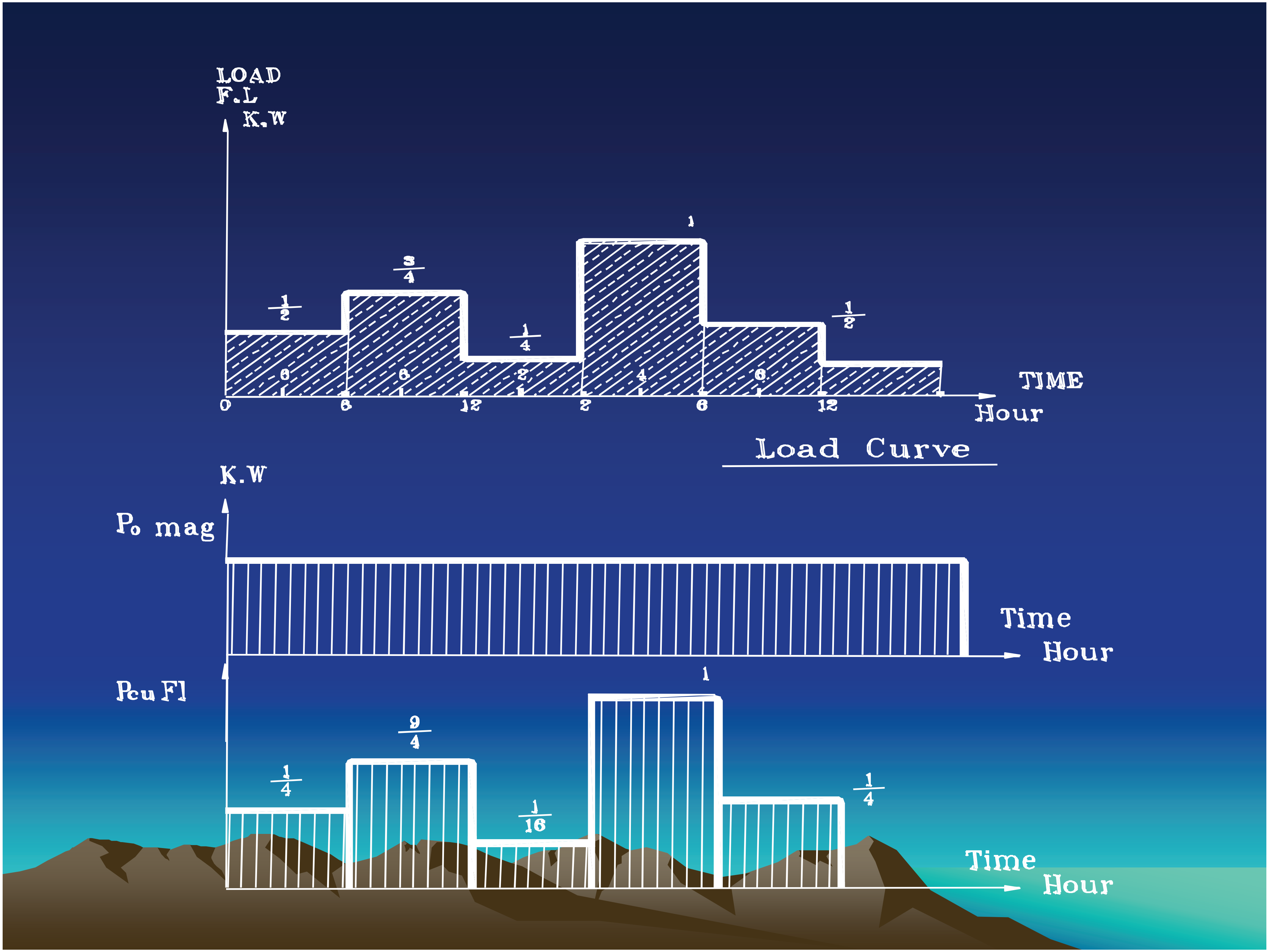
$$\eta = \frac{P_{out}}{P_{out} + \text{const.loss} + cu\ \text{loss}} \qquad = \frac{(K.V.A)_{out}\cos\phi}{(K.V.A)_{out}\cos\phi + P_o + P_{cu}}$$

وعموما الكفاءة عند أى حمل (η) تحسب كالأتى :

$$= \frac{(K.V.A)_{out}\cos\phi}{(K.V.A)_{out}\cos\phi + P_o + P_{cu}}$$

$$\eta = \frac{X\,(K.V.A)_{f.l}\cos\phi}{X\,(K.V.A)_{f.l}\cos\phi + p_o + X^2\,(p_{cu})_{f.l}}$$

$$\eta = \frac{p_{in} - \text{losses}}{p_{in}} \qquad \eta_X = \frac{X\,(K.V.A)_{f.l}\cos\phi - p_o - X^2\,(p_{cu})_{f.l}}{X\,(K.V.A)_{f.l}\cos\phi}$$



LOAD
F.L
K.W
1/2
3/4
1/4
1
1/2
6
6
2
4
6
0
6
12
2
6
12
TIME
Hour
Load Curve
K.W
Po mag
Time
Hour
Pcu Fl
1/4
9/4
1/16
1
1/4
Time
Hour

average efficiency for the transformer during 1 day ;

$$\eta = \frac{\text{Total output energy through 24 hours}}{\text{Total input energy through 24 hours}}$$

the input energy of the transformer through the day is equal to the Total out put + Total losses per/day .

losses are const. or magnetic (P_o) and are constant through the day .

the (electrical or cu) losses are variable according to the load (QI^2) .

E.X :

100 K.V.A lighting transformer has a full load loss of 3 K.V.A , the losses being equally devided between iron and copper . During a day the transformer operates , on full load for 3 hours , one half for 4 hours , the output being negligible for the reminder of the day calculate the all day efficiency .

Solution :

It should be noted that lighting transformers are taken to have a load p.f of unity iron losses for 24 hours = 1.5 x 24 = 36 K.W.h (const. losses)

FL.cu losses = 1.5 K.W

Cu loss for 3 hours on F.L = 1.5 x 3 = 4.5 k.w.h

Cu loss for half F.L = 1.5 /4 k.w.h

Cu loss for 4 hours at half the load = (1.5 / 4) x 4 = 1.5 k.w.h

Total losses = 36 + 4.5 + 1.5 = 42 k.w.h

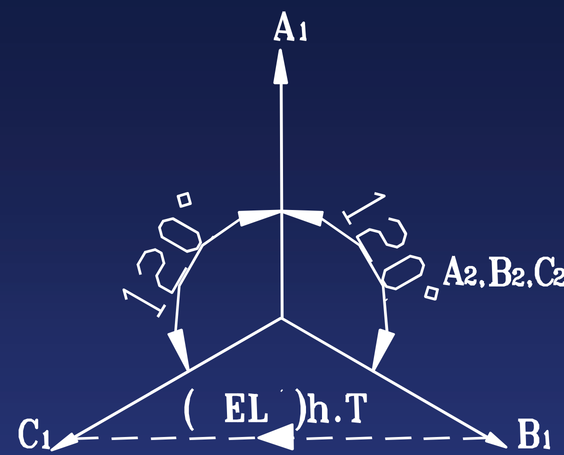
Total output = (100 x3) + (50 x 4) = 500 k.w.h

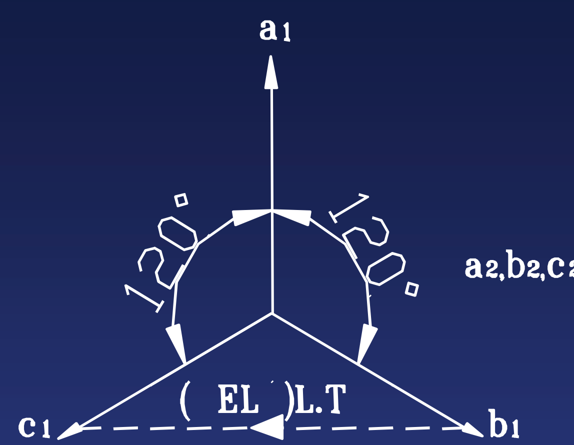
η all day = 500 x 100 / 542 = 92.26 %

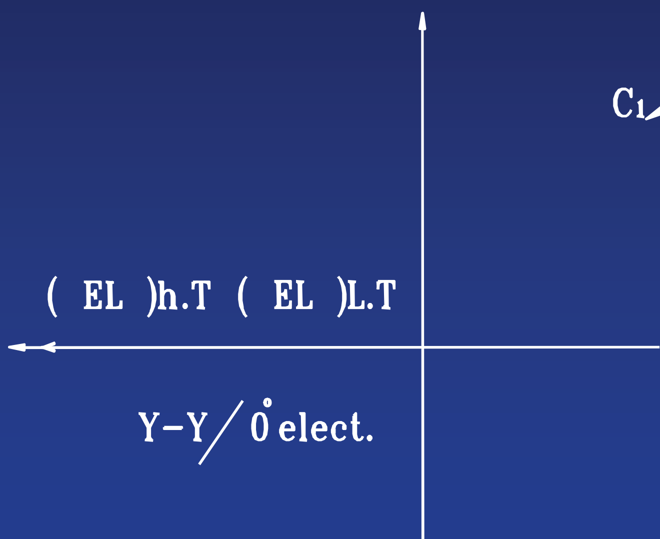
Group numbers :

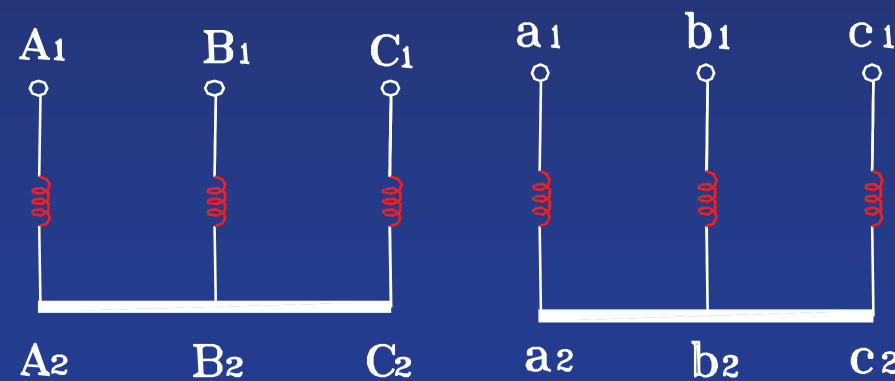
The group number indicates the phase differience between primary and secondary (H.T and L.T) line voltages in electrical degrees . It is sometimes determined as a clock reading each hour is equivalent to 30° phase difference .

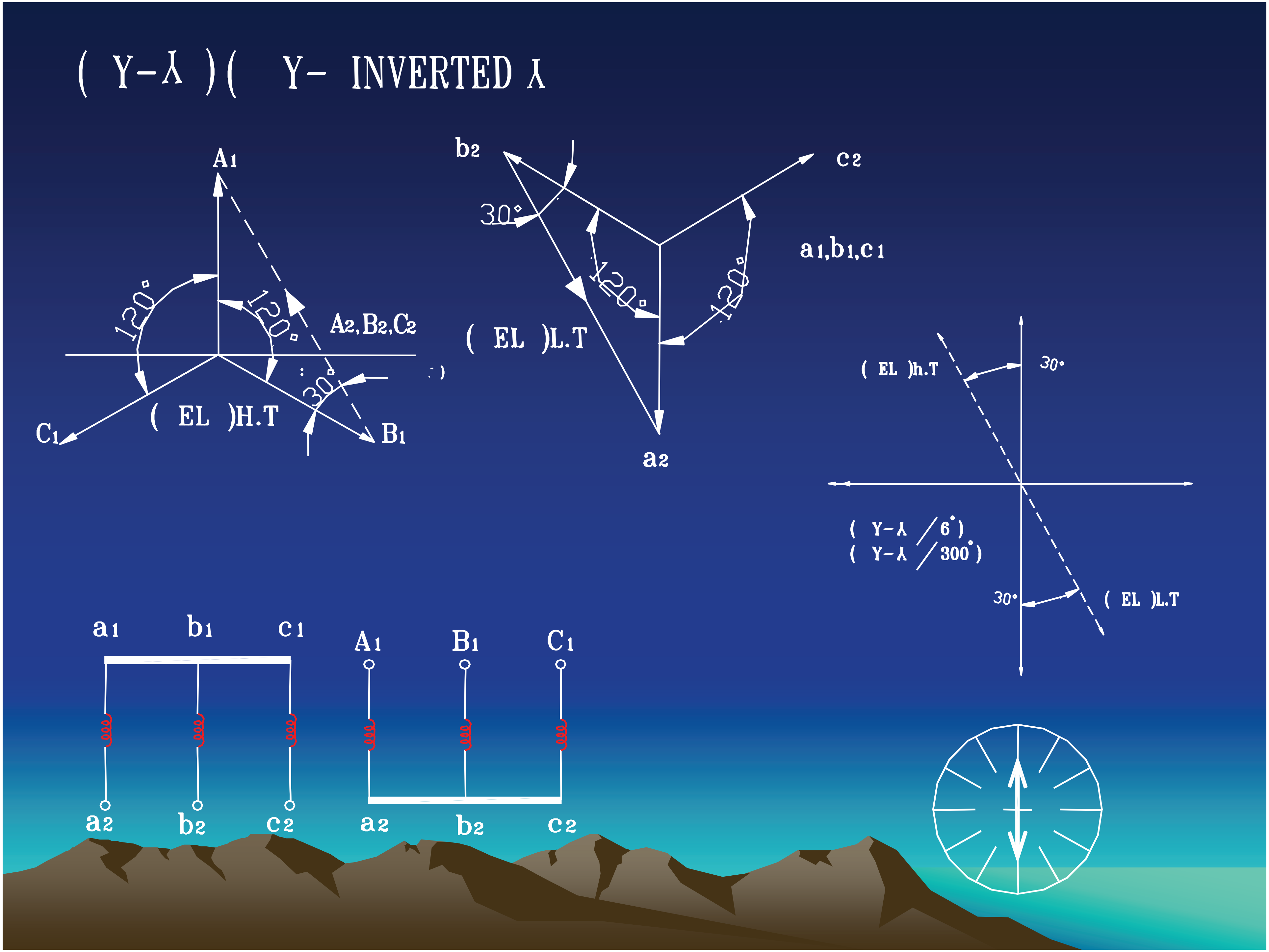
Y-Y connectios :



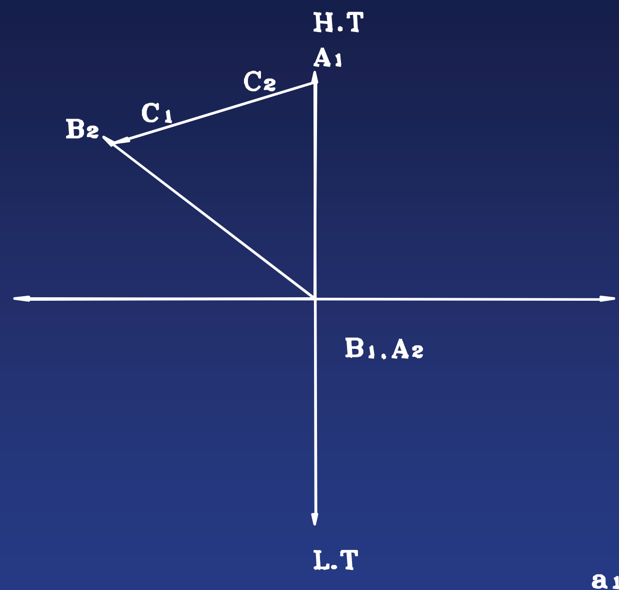


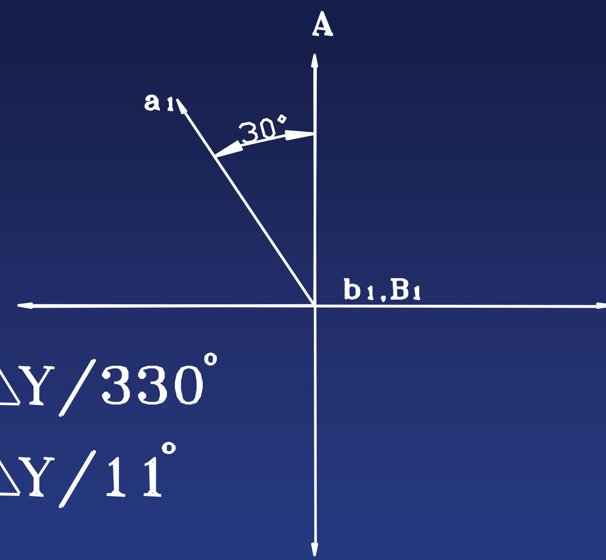





(Y-λ)(Y- INVERTED λ
A1
B1
C1
A2,B2,C2
120°
30°
(EL)H.T
b2
c2
a2
a1,b1,c1
(EL)L.T
(EL)h.T
(Y-λ / 6°)
(Y-λ / 300°)
a1 b1 c1
a2 b2 c2
A1 B1 C1
a2 b2 c2

(ΔY)

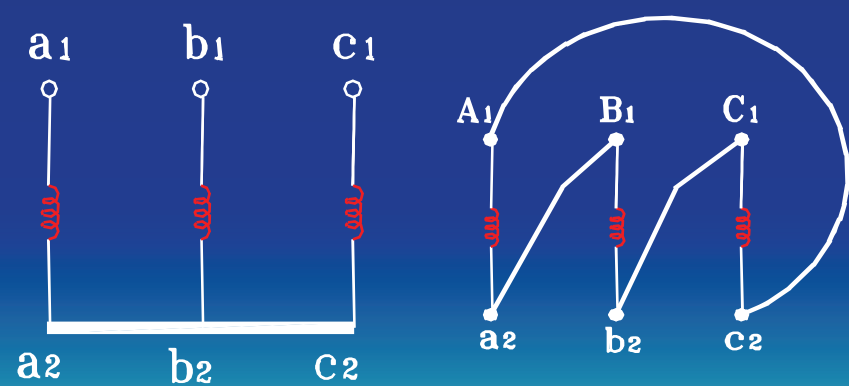




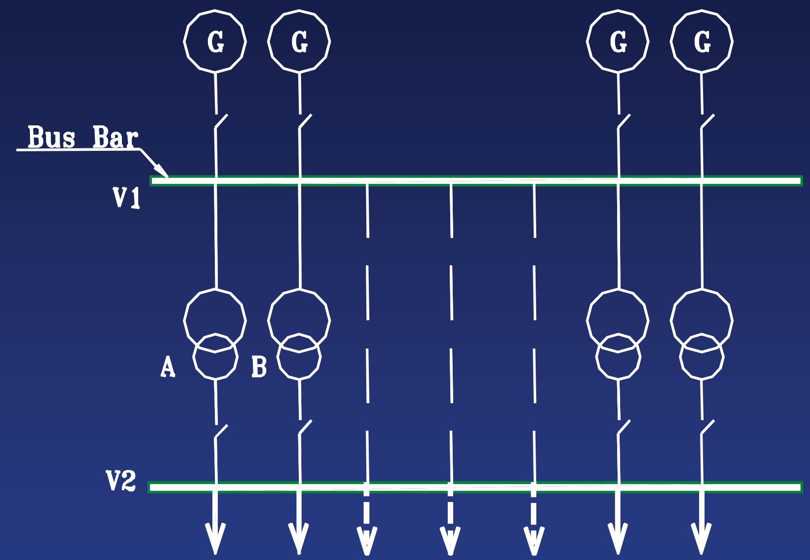
$\Delta Y/330^\circ$

$\Delta Y/11^\circ$

Y/11



Parallel operation :



In power station transformers are usually working in parallel in order to enable the connections or disconnection of any number of them according to their required load :-
The following conditions must be fulfilled for correct parallel operation :

1. the transformations ratios must be the same
2. the group number must be the same
3. the phase connection must be in same sequence
4. short circuit impendence (Zeq) must be the same